

VARIETADES EM \mathbb{R}^n

$$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^m, \quad m < n$$

$$\begin{matrix} (x, y) \\ \uparrow \quad \uparrow \\ \text{var. livres} \quad \text{var. dependentes} \end{matrix}$$

$$\det D_y F(\cdot) \neq 0$$

$$F: \mathbb{R}^n \rightarrow \mathbb{R}^m, C^1, \boxed{\text{Car } D F(x) = m}$$

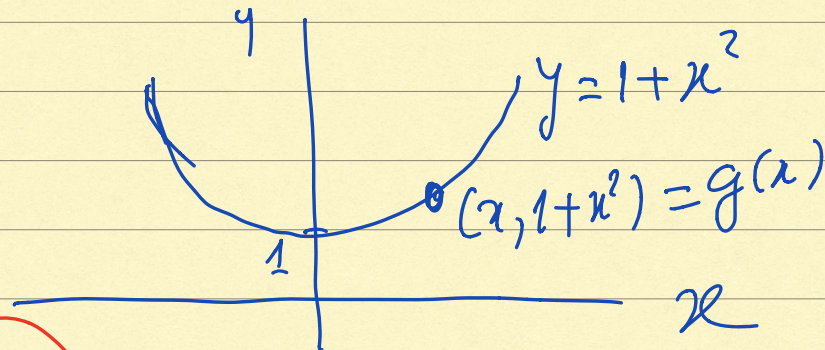
$$M = \{ (x, y) : F(x, y) = 0 \} \left\{ \begin{array}{l} \text{conjunto de} \\ \text{nível zero} \\ \text{de } F \end{array} \right.$$

$$= \{ (x, y) : y = f(x) \} \quad \text{gráfico de } f_{C^1}$$

$$= \{ (x, f(x)) \} = \{ g(x) \}$$

$$g(x) = (x, f(x)) \quad \text{imagem de } g$$

Exemplo: $M = \{(x, y) \in \mathbb{R}^2 : y - x^2 = 1\}$



$F(x, y) = y - x^2 - 1 = 0$ $F: \mathbb{R}^2 \rightarrow \mathbb{R}, \mathbb{C}^1$

$(\Rightarrow) y = 1 + x^2 = f(x)$

$g(x) = (x, f(x)) = (x, 1 + x^2)$

$DF(x, y) = \begin{bmatrix} -2x & 1 \end{bmatrix} \rightarrow \boxed{\text{vetor normal}} \neq 0$

$M \equiv$ conjunto de nível de F

\equiv gráfico de f

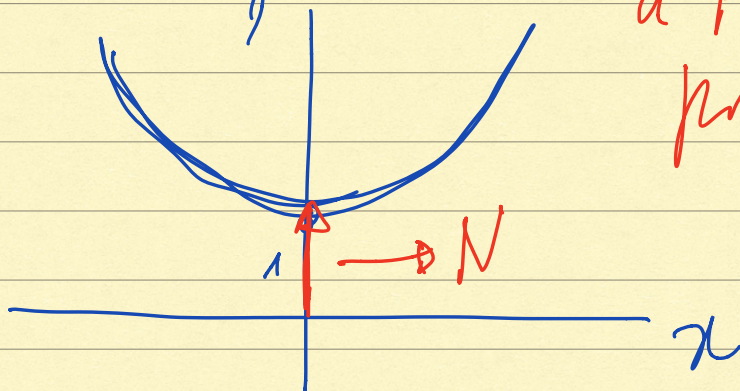
\equiv imagem de g .

$$F(0,1) = 0$$

$$y = 1 + x^2$$

$$DF(0,1) = \begin{bmatrix} 0 & \textcircled{1} \end{bmatrix} = N = \text{vector normal}$$

a to the
point $(0,1)$.



$$g(x) = (\underline{\hat{x}}, 1 + x^2), \quad x \in \mathbb{R}$$

$$g: \mathbb{R} \rightarrow \mathbb{R}^2, \quad \textcircled{C^1}$$

injectiva.

$$x_1 \neq x_2, \quad g(x_1) \neq g(x_2)$$

$$(\underline{x_1}, 1 + x_1^2) \neq (\underline{x_2}, 1 + x_2^2)$$

$$Dg(x) = \begin{bmatrix} \textcircled{1} \\ 2x \end{bmatrix} \neq 0$$

car $Dg(x) = 1$. (coluna é
lin. independente)

Se $g: \mathbb{R}^{n-m} \rightarrow \mathbb{R}^n, C^1, m < n$
injectiva, car $Dg(\cdot) = n-m$

di) se por g é uma parametri-
zação de M .

$\mathbb{R}^{n-m} \ni x \longmapsto g(x) \in \mathbb{R}^n$
 \uparrow
Var. livres. (parâmetros)

$$F(x, y) = y - x^2 - 4 = 0$$

$$\Leftrightarrow y = 1 + x^2 = f(x)$$

$$g(x) = (x, 1+x^2)$$

$$M = \{(x, y) : y - x^2 - 1 = 0\} \quad \textcircled{F}$$

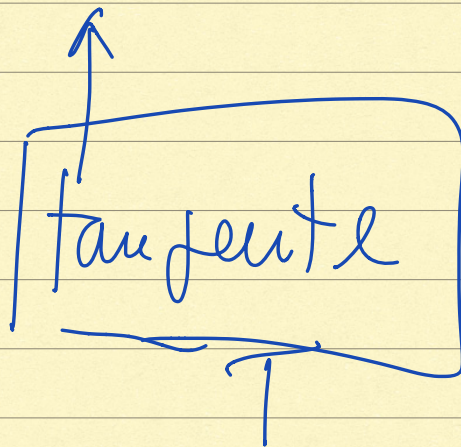
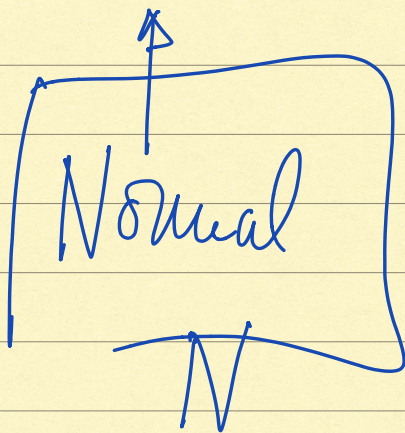
$$= \{(x, y) : y = 1+x^2\} \quad \checkmark \quad \textcircled{F}$$

$$= \{(x, 1+x^2), x \in \mathbb{R}\} \quad \textcircled{g}$$

$$F(x, y) = F(\sqrt{1+x^2})$$

$$= F(g(x)) = 0 \text{ composite!}$$

$$\underline{DF(g(x))} \cdot Dg(x) = 0$$



$$N \cdot T = 0$$

← " →

an special:

$$1) F(x) = 0$$

$$F: \mathbb{R}^n \rightarrow \mathbb{R}^m, \mathbb{C}^1$$

$$DF(\cdot) = \left[\equiv \right]_{n \times n} \rightarrow N$$

$$2) g: \mathbb{R}^{(n-m)} \rightarrow \mathbb{R}^{(n)}, \mathbb{C}^1, \text{ injective}$$

$$Dg(\cdot) = \left[\begin{array}{c} | \\ | \\ | \\ | \end{array} \right]_{n \times (n-m)}$$

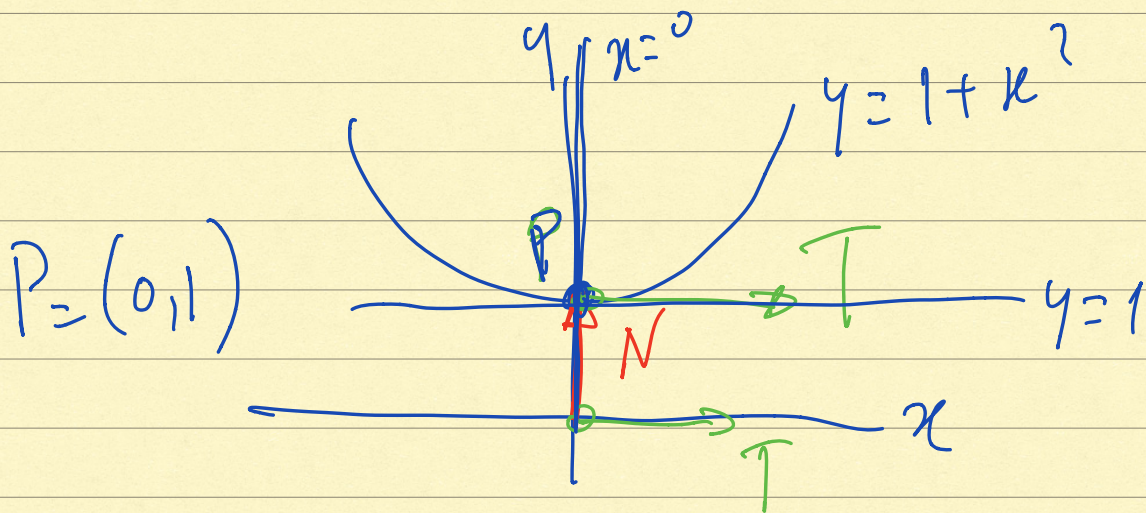
↑
T

$$\text{Ex. } F(x, y) = y - x^2 - 1 = 0$$

$$\nabla F(x, y) = (-2x, 1) = N$$

$$g(x) = (x, 1+x^2)$$

$$Dg(x) = \begin{bmatrix} 1 \\ 2x \end{bmatrix} \equiv (1, 2x) = T$$



$$\nabla F(0, 1) = (0, 1) = N$$

$$Dg(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = (1, 0) = T$$

Eq. de reta normal a M
em P . :

$$\left((x, y) - (0, 1) \right) \cdot T = 0$$

$$(x, y-1) \cdot (1, 0) = 0$$

$$\boxed{x = 0}$$

Eq. de reta tangente a M em P :

$$\left((x, y) - (0, 1) \right) \cdot N = 0$$

$$(x, y-1) \cdot (0, 1) = 0$$

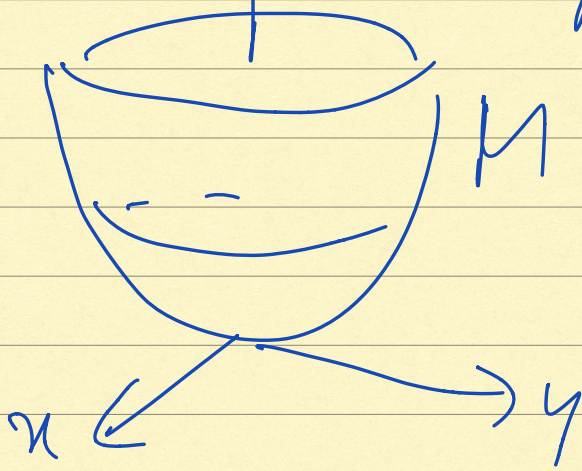
$$y-1 = 0$$

$$\boxed{y = 1}$$

Exemplo:

$$M = \{ (x, y, z) \in \mathbb{R}^3 : z = x^2 + y^2 \}$$

paraboloid



variety - 2
↓
2 parameters

1) $F(x, y, z) = z - x^2 - y^2 = 0$

$DF(x, y, z) = [-2x \quad -2y \quad \textcircled{1}] \rightarrow N$
 $\neq 0$

2) $f(x, y) = x^2 + y^2$ ✓

3) $g(x, y) = (x, y, x^2 + y^2)$ ✓

$$Dg(x, y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2x & 2y \end{bmatrix} \rightarrow \det = 1 \neq 0$$

Wunsch lin. ind.

Tangenten

$$P = (0, 0, 0)$$

$$N = (0, 0, 1) \quad \text{Normal} \rightarrow \text{vekt. normal}$$

$$\left. \begin{array}{l} T_1 = (1, 0, 0) \\ T_2 = (0, 1, 0) \end{array} \right\} \text{tangenten}$$

↓
plano
tangente.

$$N \cdot T = 0$$

Eq. de recte normal a M en P :

$$P = (0, 0, 0) \quad X = (x, y, z)$$

$$\begin{cases} (X - P) \cdot T_1 = 0 \\ (X - P) \cdot T_2 = 0 \end{cases}$$

Eq. do plano tangente a M em P :

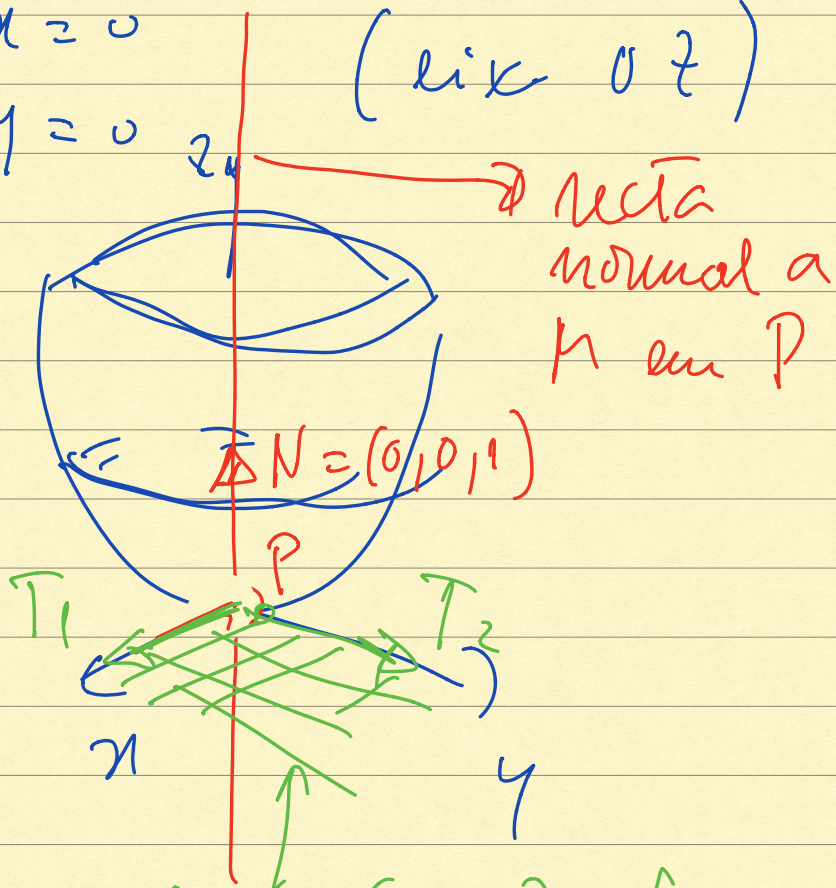
$$(X - P) \cdot N = 0$$

————— || —————

recte normal:

$$\begin{cases} (x, y, z) \cdot (1, 0, 0) = 0 \\ (x, y, z) \cdot (0, 1, 0) = 0 \end{cases}$$

$$(\Rightarrow) \begin{cases} x=0 \\ y=0 \end{cases} \quad (\text{eix } Oz)$$



plano tangente: $(x-P) \cdot N = 0$

$$(x, y, z) \cdot (0, 0, 1) = 0$$

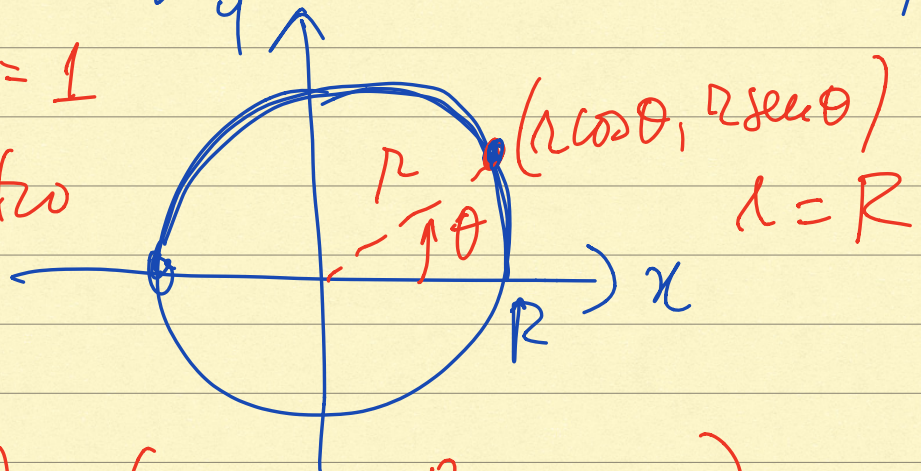
$$\boxed{z = 0} \quad \checkmark$$

Parametrizar:

Ex $M = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = R^2\}$

$\dim(M) = 1$

1 parámetro



$$g(\theta) = (R \cos \theta, R \text{sen} \theta)$$

$$0 < \theta < 2\pi$$

Ex: $M: \boxed{x^2 + y^2 = 1}; 0 < z < 1$

$\dim(M) = 2$ $\boxed{\rho = 1}$ ρ

$$g(\theta, z) = (\cos \theta, \text{sen} \theta, z)$$